Cross-market High-dimensional Nonstationary Coupling Learning

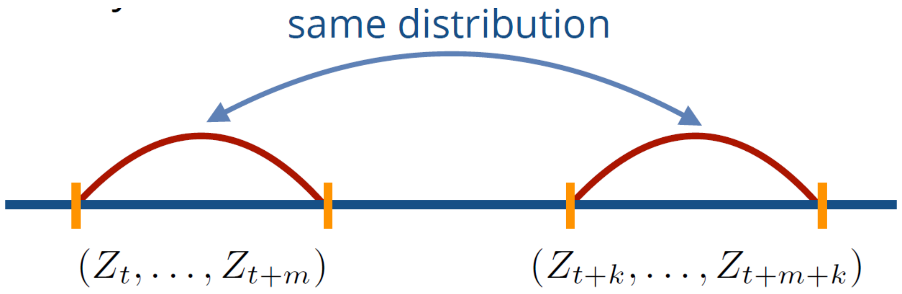
Longbing Cao

The Data Science Lab
University of Technology Sydney, Australia

www.datasciences.org

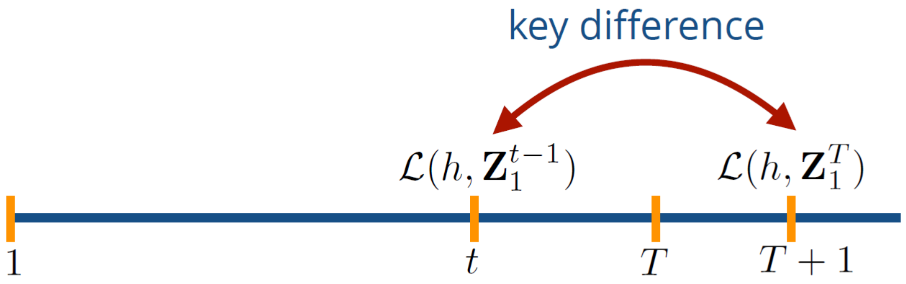
Nonstationarity

- Stationarity

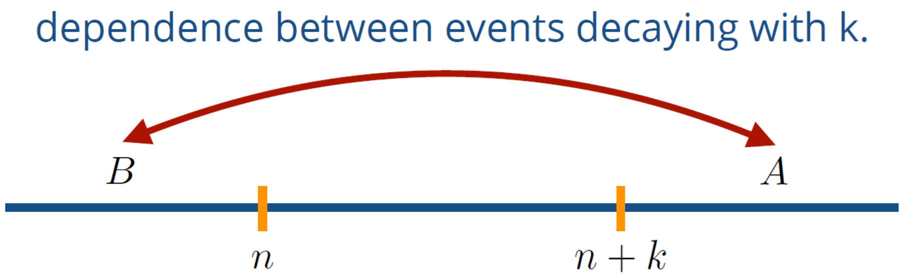


NIPS16 tutorial: Theory and algorithms for forecasting non-stationary time series

- Weak/non-stationarity



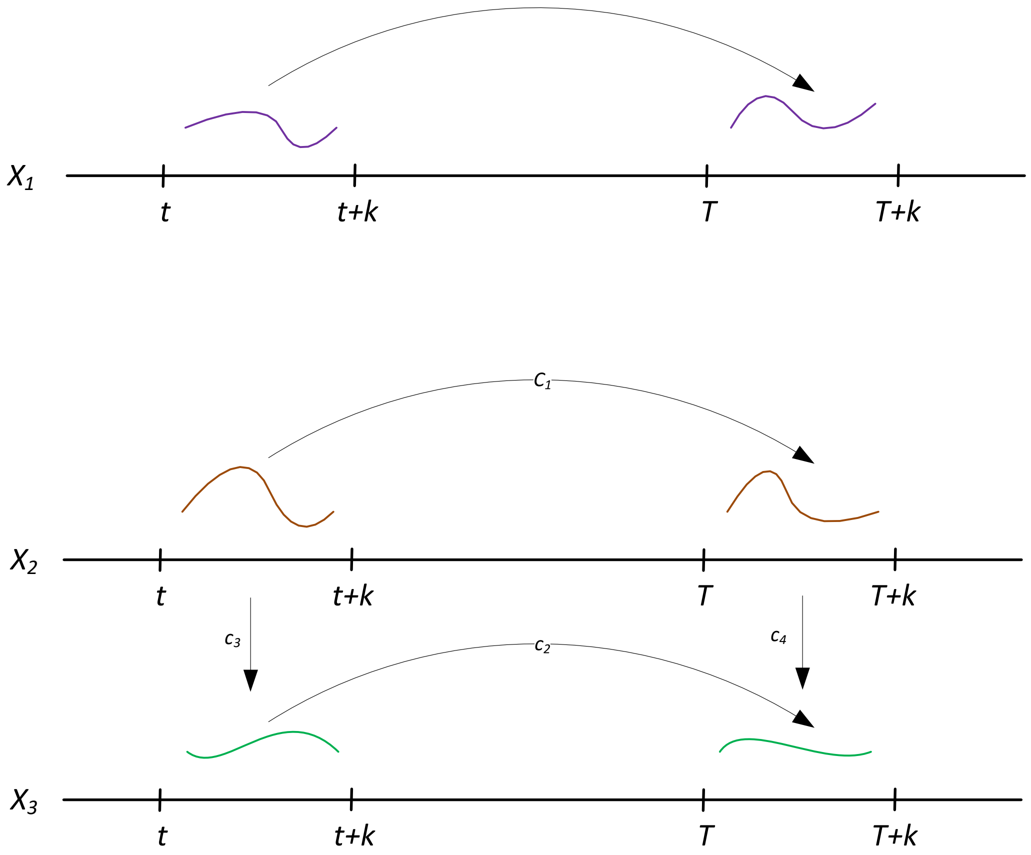
$$\Delta(\mathbf{q}) = \sup_{h \in H} \left| \mathcal{L}(h, \mathbf{Z}_1^T) - \sum_{t=1}^{T} q_t \, \mathcal{L}(h, \mathbf{Z}_1^{t-1}) \right|$$



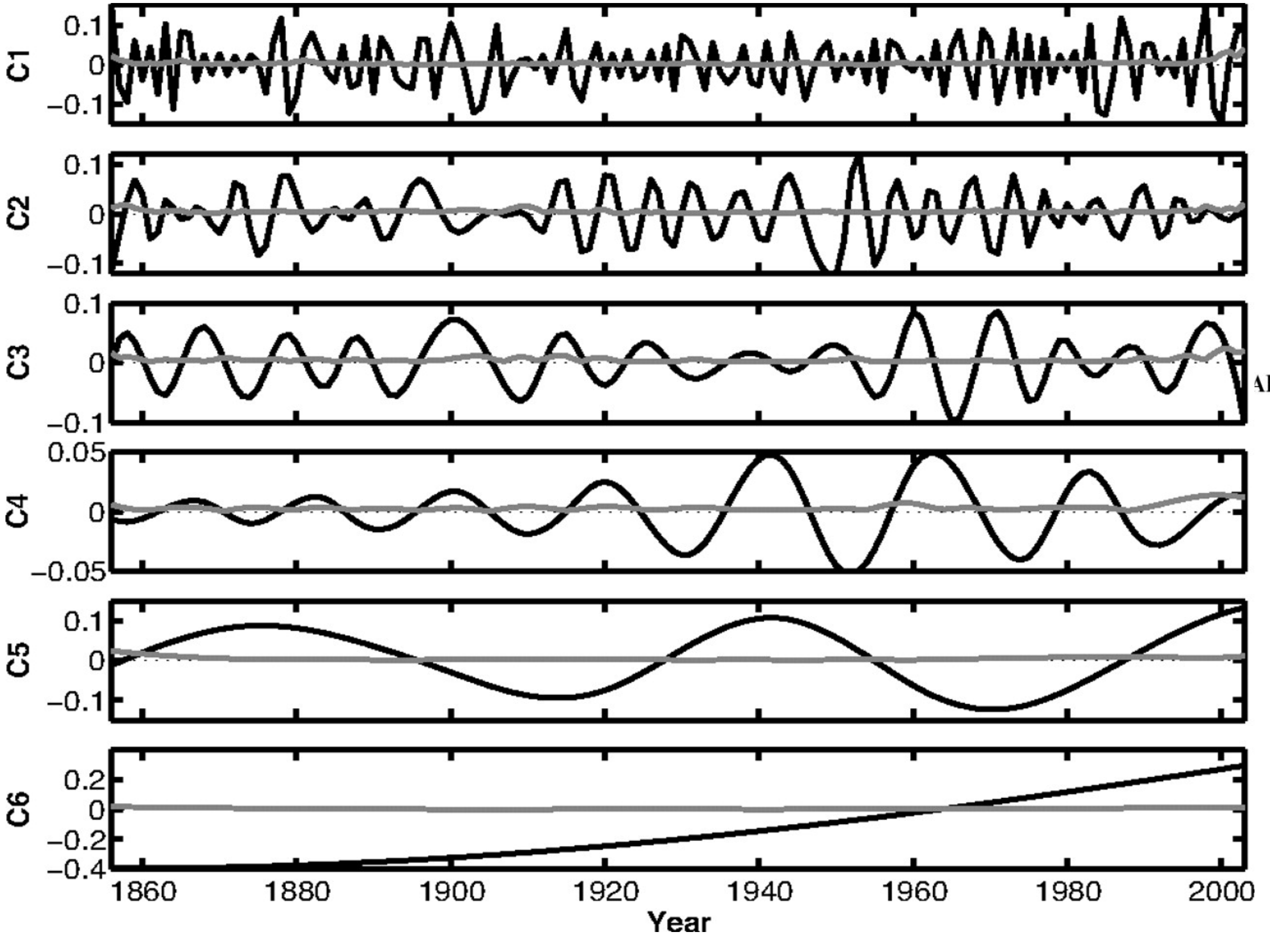
$$\beta(k) = \sup_n \mathop{\mathbb{E}}_{B \in \sigma_{-\infty}^{n}} \left[\sup_{A \in \sigma_{n+k}^{\infty}} \Big| \mathbb{P}[A \mid B] - \mathbb{P}[A] \Big| \right] \to 0$$

Nonstationary couplings

- The change of one nonstationary variable is coupled with the change of another one
- Couplings: Association, correlation, dependence, causality, latent relations, etc.



Nonstationary heterogeneous couplings



Z Wu, etc. (2007) On the trend, detrending, and variability of nonlinear and nonstationary time series

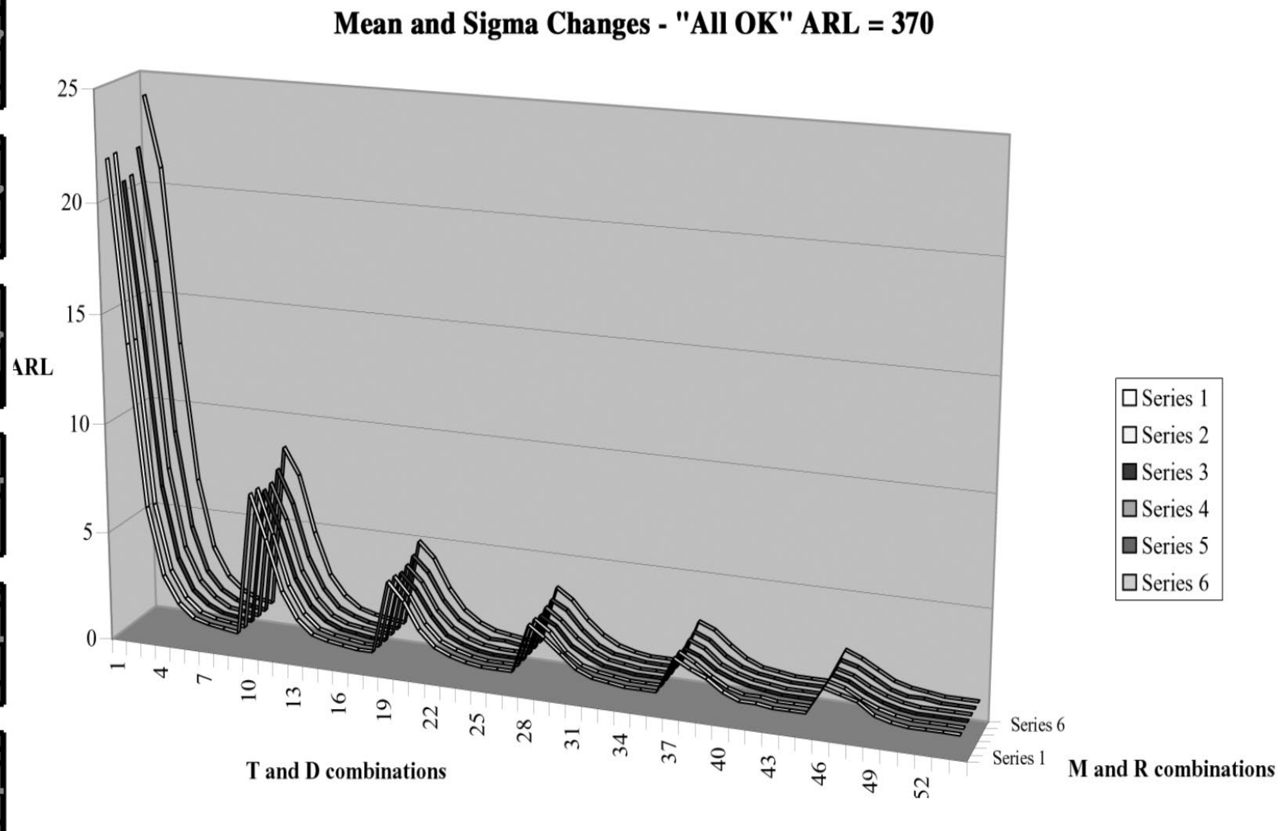


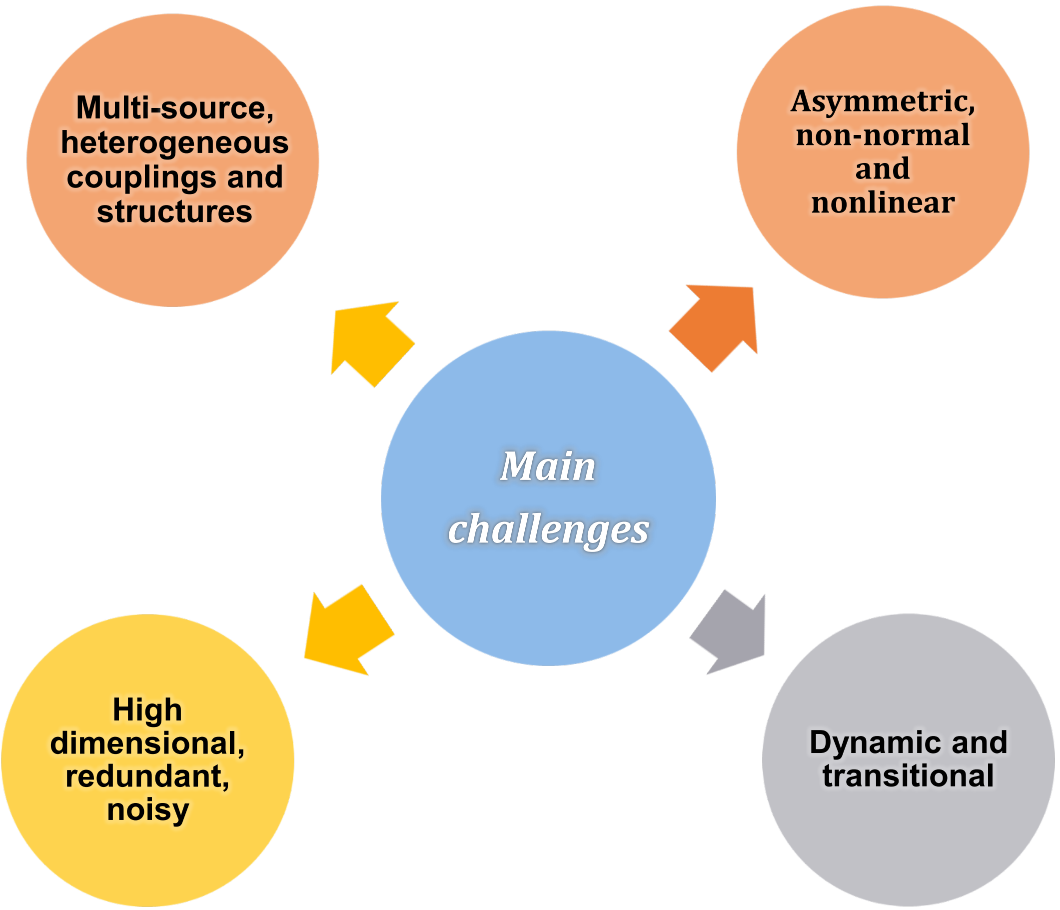
Figure 3. Plotted ARL values of Table 3.

D. Rahardja (2005) X-Charts versus X / MR Chart Combinations: IID Cases and Non-IID Cases

Main challenges

Financial variables are heterogeneous, coupled w.r.t. various structures

Cross market dependence is often embedded with nonlinear, non-normal and asymmetric dependences



Generally more than 20 financial variables leading to over ten thousand features for a group of time series in a time window

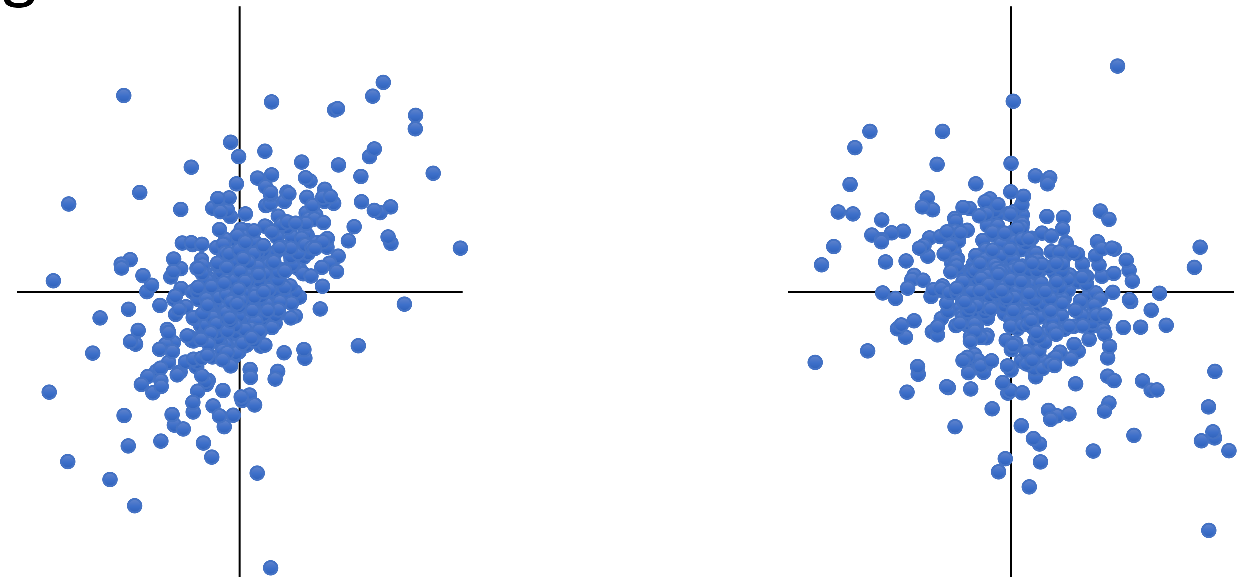
Financial variables and their influence across markets are dynamic/ volatile, transitional

Example 1:
Copula-based Dependence Structure Modeling

J Xu, et al. Copula-Based High Dimensional Cross-Market Dependence Modeling
W Wei et al. Modeling Asymmetry and Tail Dependence among Multiple Variables by Using Partial Regular Vine
W Wei et al. Model the Complex Dependence Structures of Financial Variables by Using Canonical Vine

An example

Positive and negative correlations

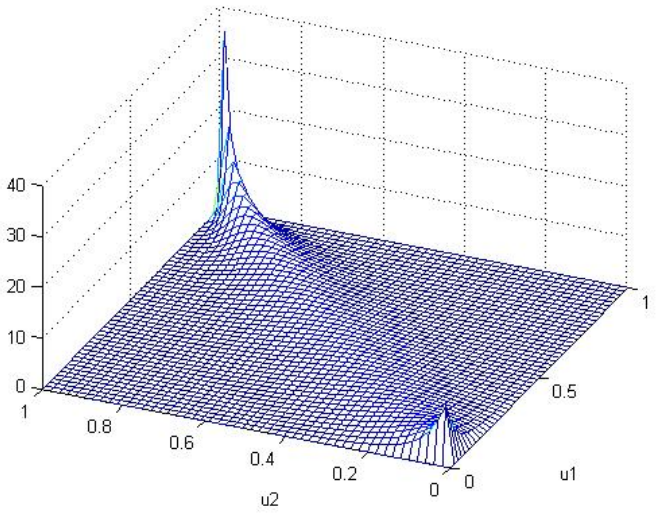


a. FTSE100 and S&P500 ***b. FTSE100 and GBP***

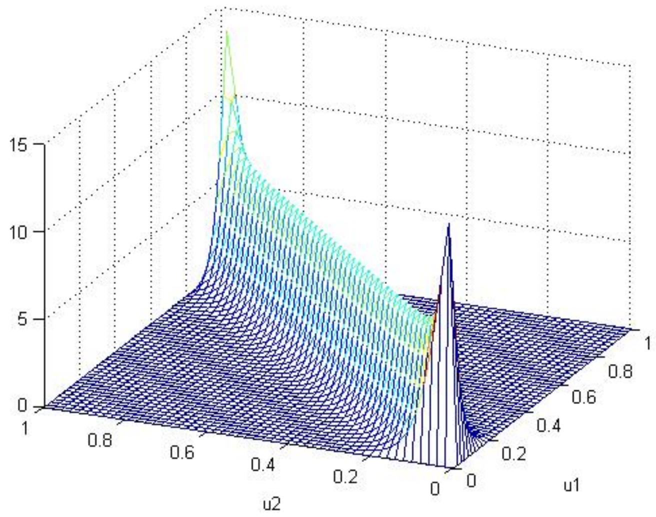
- Stock markets and exchange rate markets are dependent
- Dependences may be diverse
- Cross-market couplings have to be considered in multivariate modeling

An example

Asymmetric and tail dependence



***Gumbel Copula (ϑ = 3.35)* between S&P500 and STOXX50E**



Frank Copula (ϑ = 28.5)* between *FTSE100 and GBP

Daily returns of S&P 500 and EUR/USD (01/01/2008 – 31/12/2010)

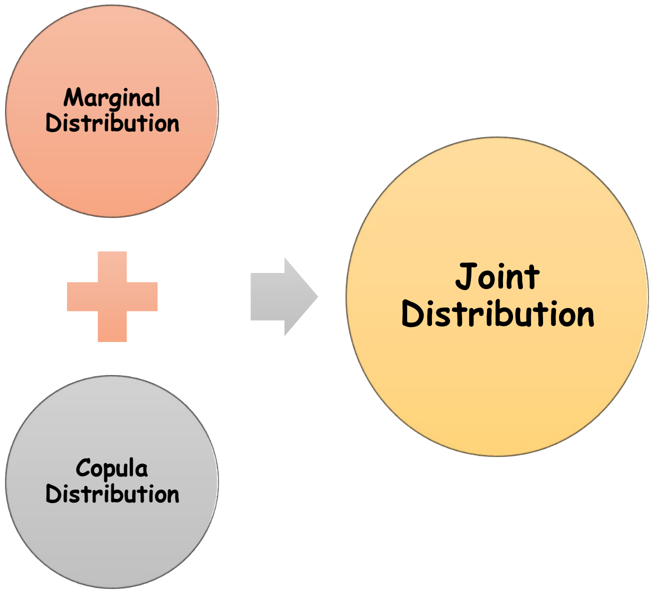
W Wei et al. Modeling Asymmetry and Tail Dependence among Multiple Variables by Using Partial Regular Vine

Challenges

- Multiple heterogeneous financial variables
- Stylized fact: fat tail and asymmetric correlations
- Dependence strength and structure

Copula-based dependence modeling

- Modeling joint distribution between a group of random variables



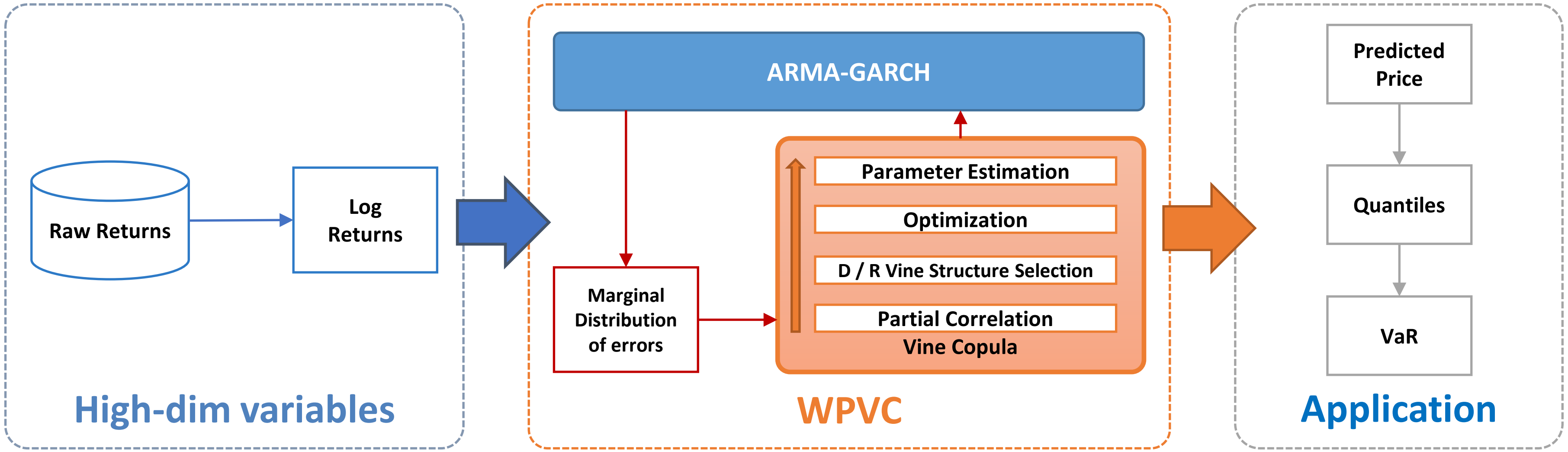
$$F_1(x_1, x_2, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))$$

$$C(u_1, u_2, \ldots, u_n) = F(F_1^{-1}(u_1), F_2^{-1}(u_2), \ldots, F_n^{-1}(u_n))$$

$$f(x_1, x_2, \ldots, x_n) = c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) \prod_{i=1} f_i(x_i)$$

Partial vine for structural dependence

- Weighted partial vine copula (WPVC)



- Partial D vine tree structure for asymmetric dependence in tail risk
- Bivariate copula with different types of tail dependencies
- Truncation with conditional independence (vs. correlations) for high-dimensional

Partial correlation

- Partial D vine structure

$$\rho_{1,2;3,\ldots,n} = \frac{\rho_{1,2;3,\ldots,n-1} - \rho_{1,n;3,\ldots,n-1} \cdot \rho_{2,n;3,\ldots,n-1}}{\sqrt{1-\rho^2_{1,n;3,\ldots,n-1}} \cdot \sqrt{1-\rho^2_{2,n;3,\ldots,n-1}}}$$

- Lower and upper tail dependence coefficients

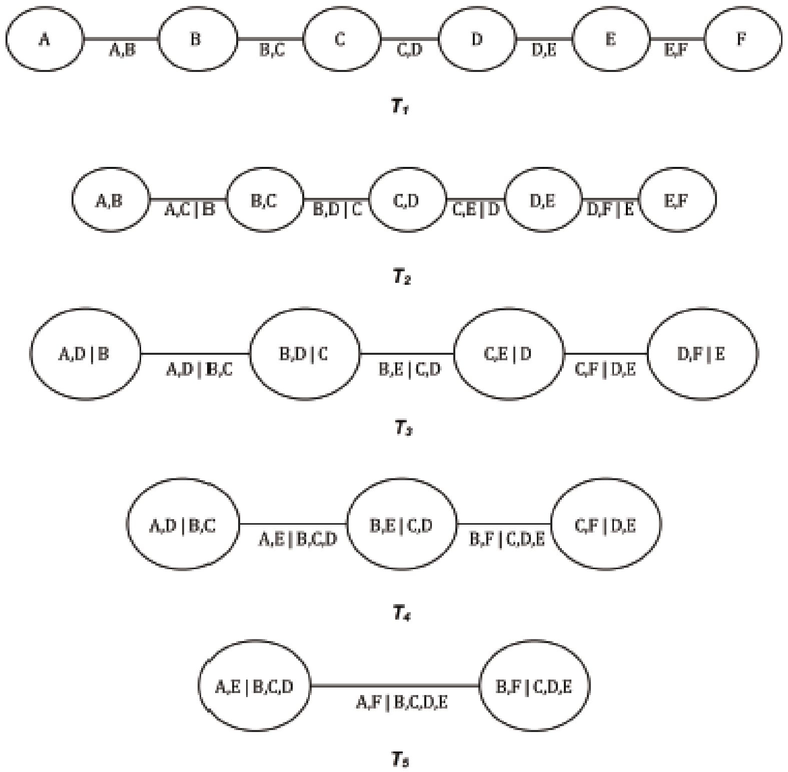
$$\begin{aligned}
\lambda_L &= \lim_{u\to 0} Pr\{U_1 \le u, \ldots, U_n \le u \mid U_n \le u\} \\
&= \lim_{u\to 0} \frac{C(u, \ldots, u)}{u} \\
\lambda_U &= \lim_{u\to 0} Pr\{U_1 > 1-u, \ldots, U_n > 1-u \mid U_n > 1-u\} \\
&= \lim_{u\to 0} \frac{\overline{C}(1-u, \ldots, 1-u)}{u}
\end{aligned}$$

- *0*
- $\in$ (0, 1

Partial D vine tree construction

Weighted partial D vine tree construction

- 6 variables, 20 partial correlations
- The smallest PC as the edge of T5: {A,F;B,C,D,E}, conditioned set {A,F} and conditioning set {B,C,D,E}
- Nodes in T5:
  - Constraint sets {A,B,C,D,E} and {F,B,C,D,E}
  - Find the smallest PC for each constraint set and treat them as nodes, e.g. {A,E;B,C,D} and {B,F;C,D,E}
- T4, T3
- Best D vine:



$$\operatorname{argmax}(-ln(D)) \qquad D = \prod_{i,j} (1 - W_i \rho^2_{i,j;d(i,j)})$$

$$W(h) = \begin{cases} 0.5 \times \frac{e^{-m_0(k-h)}}{\sum_{i=1}^{k} e^{-m_0(k-i)}}, & h \in [1, k]; \\ 0.5 \times \frac{e^{-m_0(h-k)}}{\sum_{i=k+1}^{N-1} e^{-m_0(i-k)}}, & h \in (k, N-1]. \end{cases}$$

Parameter estimation

- Maximum Log-Likelihood estimation to estimate the parameters of copula constructed w.r.t. vine structures

$$L(\xi : x) = \sum_{j=1}^{n} \left\{ \sum_{i=1}^{p} \ln f_i\left(x_{i,j}; \phi_i\right) + \ln c\left(F_1(x_1, n), F_1(x_2, n) \cdots, F_p\left(x_p, n\right); \theta\right) \right\}$$

$$\xi = (\phi_1, \dots \phi_p, \theta)$$

Estimate parameters for marginal distributions:

$$L_m(\phi : x) = \sum_{i=1}^{p} \sum_{j=1}^{n} ln f_i(x_{i,j}; \phi_i) \qquad \hat{\phi} = \underset{\phi}{\operatorname{argmax}}\, L_m(\phi : x)$$

Estimate parameter in copula:

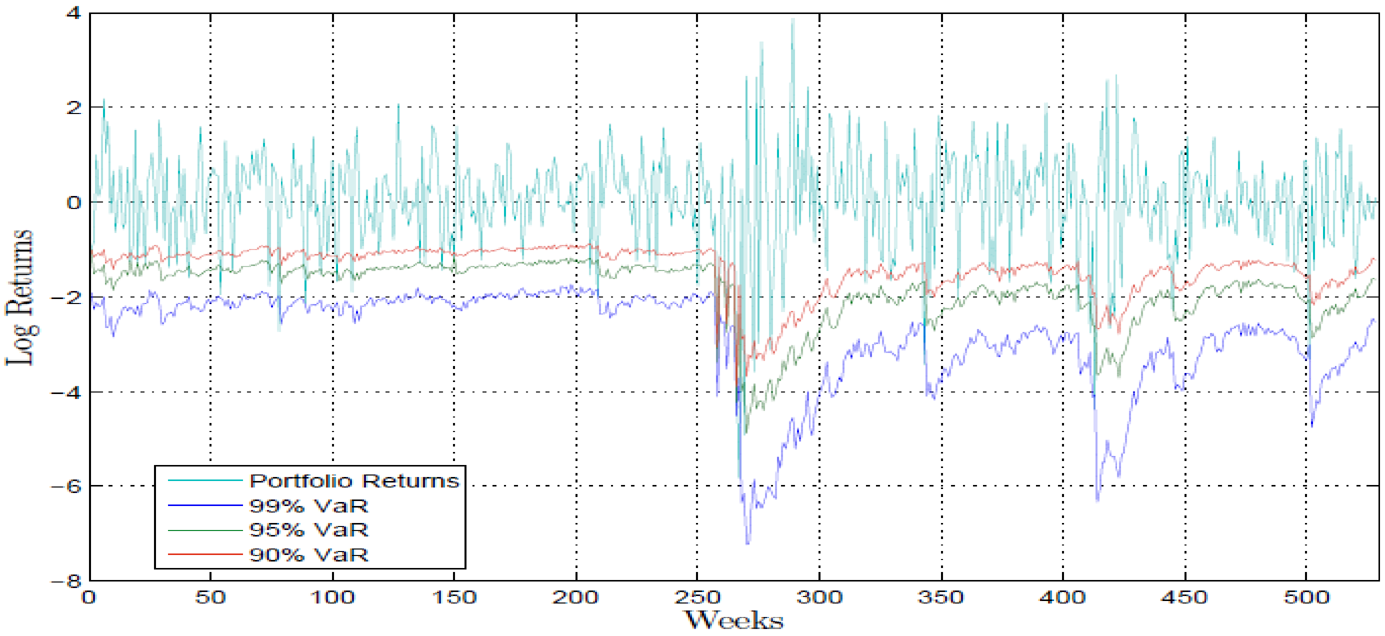
$$L_c(\theta; u, \phi) = \sum_{i=1}^{p} ln(c(F_1(x_1, n), \dots, F_p(x_p, n); \theta)) \qquad \hat{\theta} = \underset{\theta}{\operatorname{argmax}}\, L_c(\theta; u, \phi)$$

Case study

- Backtesting VaR – Log-likelihood ratio

	$1-\alpha$	POF[1]	LR_{UC}[2]	LR_{IC}[2]	LR_{CC}[2]
$WPVC_{0.05}$	99%	5 1.08%	0.0324 (0.857)	2.315 (0.314)	2.347 (0.143)
	95%	26 5.64%	0.382 (0.536)	0.188 (0.868)	0.570 (0.752)
	90%	52 11.50%	1.100 (0.294)	1.582 (0.254)	2.683 (0.261)
D_STD	99%	9 1.95%	2.186 (0.139)	2.221 (0.136)	4.408 (0.110)
	95%	27 5.86%	0.451 (0.730)	0.133 (0.916)	0.584 (0.618)
	90%	57 12.36%	1.363 (0.547)	3.533 (0.176)	4.896 (0.086)
D_Ken	99%	10 2.17%	3.376 (0.066)	1.843 (0.175)	5.218 (0.074)
	95%	27 5.86%	0.451 (0.730)	0.133 (0.916)	0.584 (0.618)
	90%	57 12.36%	1.363 (0.547)	3.533 (0.033)	4.896 (0.086)
$Cvine$	99%	11 2.39%	4.770 (0.029)	1.439 (0.230)	6.209 (0.045)
	95%	29 6.29%	0.662 (0.628)	0.042 (0.838)	0.704 (0.554)
	90%	59 12.80%	1.782 (0.326)	4.469 (0.035)	6.251 (0.025)
DCC	99%	103 22.34%	466.082 (0.000)	5.449 (0.021)	471.533 (0.000)
	95%	133 28.85%	276.570 (0.000)	15.257 (0.004)	291.827 (0.000)
	90%	59 32.97%	180.570 (0.000)	15.333 (0.004)	195.903 (0.000)

- VaR forecasting of portfolio return



- 25 indicators: 8 exch rates, 13 indices, 3 comdity, 1 comdty index
- In-sample: 5 years; out-of-sample: 10 years
- ARMA(1; 1)-GARCH(1; 1): stocks
- AR(1)-GARCH(1; 1): exchange rate

Example 2:
Deep modeling of financial couplings

W Cao et al. Deep Modeling Complex Couplings within Financial Markets

Market couplings

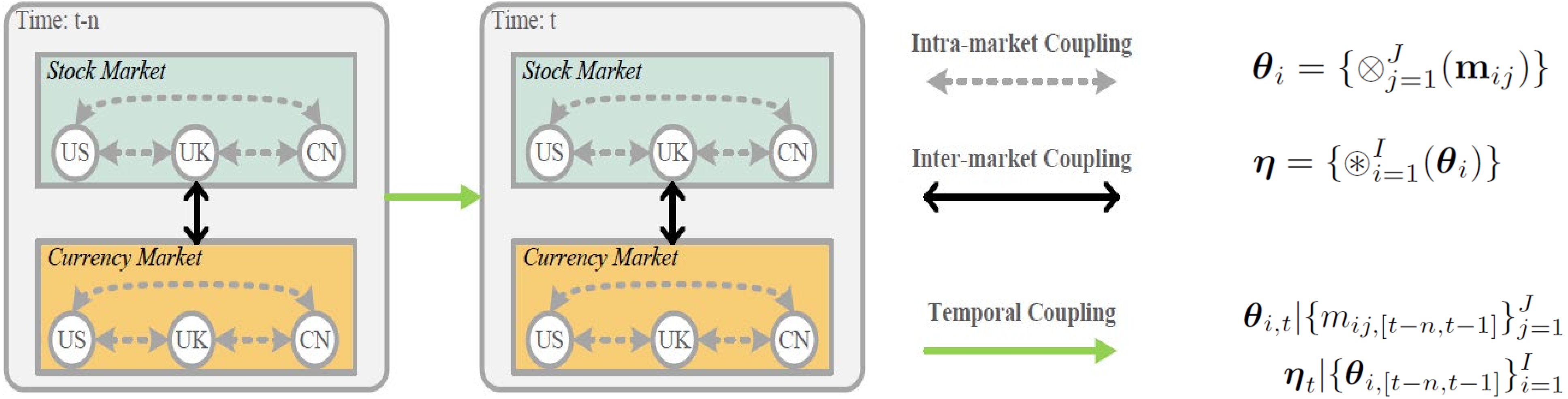


Figure 1: A demonstration of complex couplings between financial markets

Two-layer market coupling learning

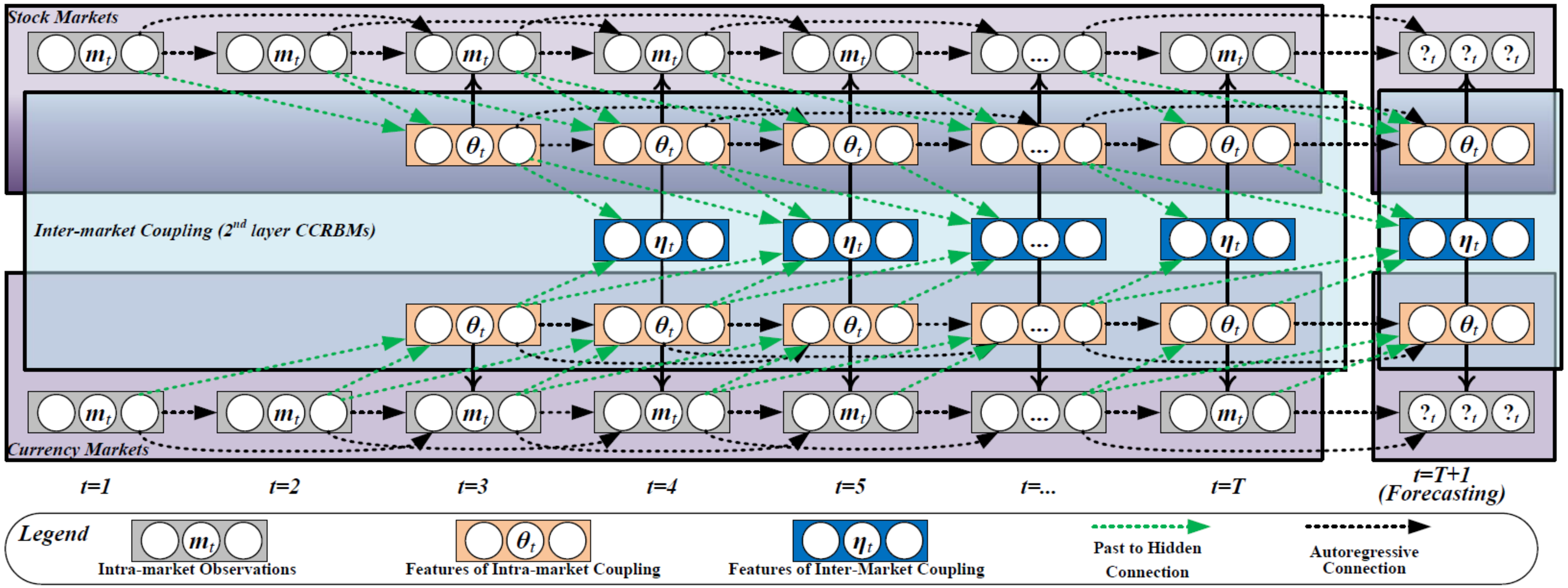
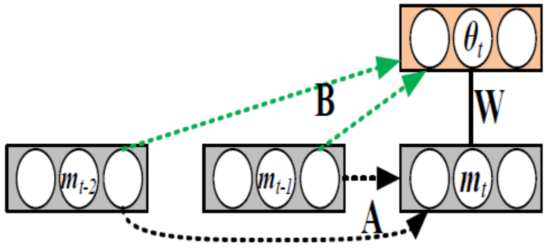


Figure 2: Modeling framework of CTDBN. Here, the demonstration shows two heterogeneous financial markets, stock and currency. The first-layer are CGRBMs to model the intra-maket couplings while CCRBMs are built on the first layer to model inter-market couplings.

Intra/inter-market couplings



(a)

Conditional Gaussian Restricted Boltzmann Machines: CGRBM intra-market couplings

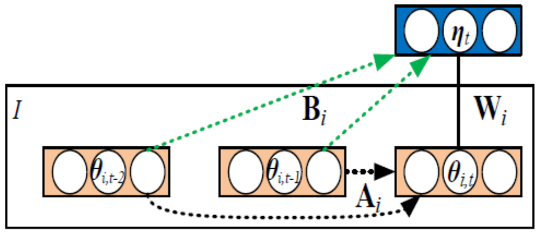
$$P(\mathbf{v},\mathbf{h}\mid\mathbf{u}) = exp(-E(\mathbf{v},\mathbf{h},\mathbf{u}))/Z$$

$$E(\mathbf{v},\mathbf{h},\mathbf{u}) =$$

$$-\frac{\mathbf{v}^{\mathbf{T}}\mathbf{W}\mathbf{h}}{\sigma} - \mathbf{u}^{\mathbf{T}}\mathbf{A}\mathbf{v} - \mathbf{u}^{\mathbf{T}}\mathbf{B}\mathbf{h} + \frac{(\mathbf{v}-\mathbf{a})^{\mathbf{T}}(\mathbf{v}-\mathbf{a})}{2\sigma^2} - \mathbf{b}^{\mathbf{T}}\mathbf{h}$$

$$P(h_f = 1 \mid \mathbf{v},\mathbf{u}) = s(b_f + \mathbf{u}^{\mathbf{T}}\mathbf{B}_{:,f} + \mathbf{v}^{\mathbf{T}}\mathbf{W}_{:,f}/\sigma)$$

$$P(v_d \mid \mathbf{v},\mathbf{u}) = \mathcal{N}(a_d + \mathbf{u}^{\mathbf{T}}\mathbf{A}_{:,d} + \sigma\mathbf{W}_{d,:}\mathbf{h}, \sigma^2)$$



(b)

Figure 3: (a) A CGRBM to model intra-market coupling at time t; (b) A CCRBM to model inter-market coupling at time t

Coupled Conditional Restricted Boltzmann Machines: CCRBM inter-market couplings

$$E(\{\boldsymbol{\theta}_{i,t}\},\boldsymbol{\eta}_t,\{\boldsymbol{\theta}_{i,<t}\}) = -\mathbf{b}^T\boldsymbol{\eta}_t - \sum_{i=1}^{I}\mathbf{a}_i^T\boldsymbol{\theta}_{i,t} -$$

$$\sum_{i=1}^{I}\boldsymbol{\theta}_{i,t}^T\mathbf{W}_i\boldsymbol{\eta}_t - \sum_{i=1}^{I}\boldsymbol{\theta}_{i,<t}^T\mathbf{A}_i\boldsymbol{\theta}_{i,t} - \sum_{i=1}^{I}\boldsymbol{\theta}_{i,<t}^T\mathbf{B}_i\boldsymbol{\eta}_t$$

$$\boldsymbol{\theta}_{i,<t} = [\boldsymbol{\theta}_{i,t-1},\boldsymbol{\theta}_{i,t-2},\ldots,\boldsymbol{\theta}_{i,t-n}]$$

$$P(\theta_{ift} = 1 \mid \boldsymbol{\eta}_{ht},\{\boldsymbol{\theta}_{i,<t}\}) =$$

$$s(\mathbf{a}_{if} + \boldsymbol{\theta}_{i,<t}^T(\mathbf{A}_i)_{:,f} + (\mathbf{W}_i)_{f,:}\boldsymbol{\eta}_t)$$

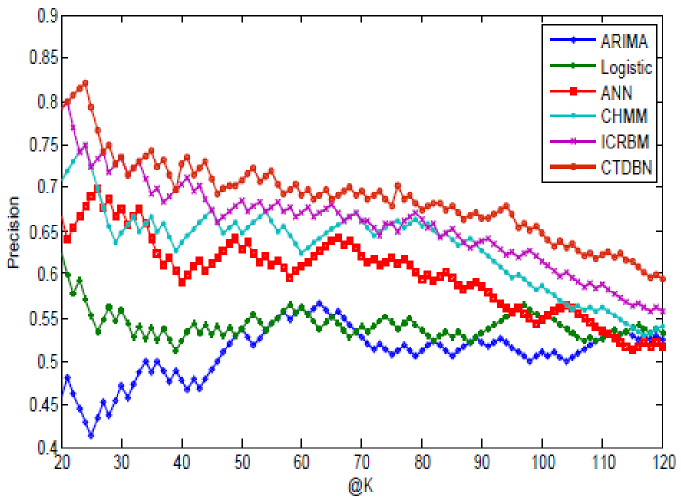
$$P(\eta_{ht} = 1 \mid \{\boldsymbol{\theta}_{i,t}\},\{\boldsymbol{\theta}_{i,<t}\}) =$$

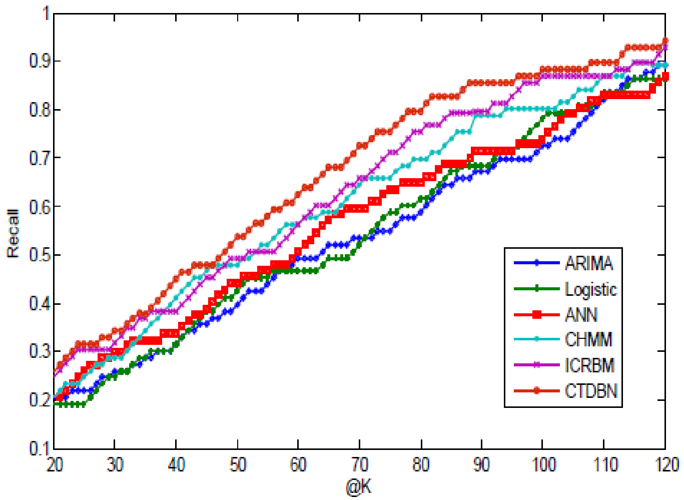
$$s(\mathbf{b}_h + \sum_{i=1}^{I}\boldsymbol{\theta}_{i,t}^T(\mathbf{W}_i)_{:,h} + \sum_{i=1}^{I}\boldsymbol{\theta}_{i,<t}^T(\mathbf{B}_i)_{:,h})$$

Return prediction

Table 2: Performance of comparative methods in US, China and India markets

Model	Accuracy						ARR					
	Stock			Currency			Stock			Currency		
	US	China	India	US	China	India	US	China	India	US	China	India
ARIMA	0.5357	0.5071	0.5029	0.5471	0.5353	0.5214	-0.1356	0.0415	-0.0675	0.1479	-0.0116	0.0304
Logistic	0.5643	0.55	0.5196	0.6	0.6059	0.5386	0.0226	0.0796	0.0558	0.0269	0.0428	0.0645
ANN	0.6	0.6	0.5752	0.6235	0.6059	0.5747	0.1217	0.1486	0.0788	0.1332	0.1244	0.1032
CHMM	0.6533	0.6214	0.5852	0.6471	0.6353	0.5709	0.1934	0.1426	0.1132	0.1645	0.1498	0.1555
CGRBM	0.6357	0.6235	0.5898	0.6565	0.64	0.5932	0.1568	0.1526	0.1410	0.1758	0.1456	0.1660
CTDBN	**0.6729**	**0.6324**	**0.6258**	**0.6734**	**0.6535**	**0.6152**	**0.2073**	**0.1682**	**0.2261**	**0.1926**	**0.1792**	**0.1972**





Example 3:
Coupled Hidden Markov Model-based Dependence Modeling

W. Cao et al. Deep Modeling Complex Couplings within Financial Markets
W. Cao, et al. Financial Crisis Forecasting via Coupled Market State Analysis
L. Cao, et al. Coupled Behavior Analysis with Applications

Challenges

- Couplings within/between financial markets
- Couplings within/between financial variables
- Heterogeneity between markets, between financial variables

Modeling within/between-market couplings

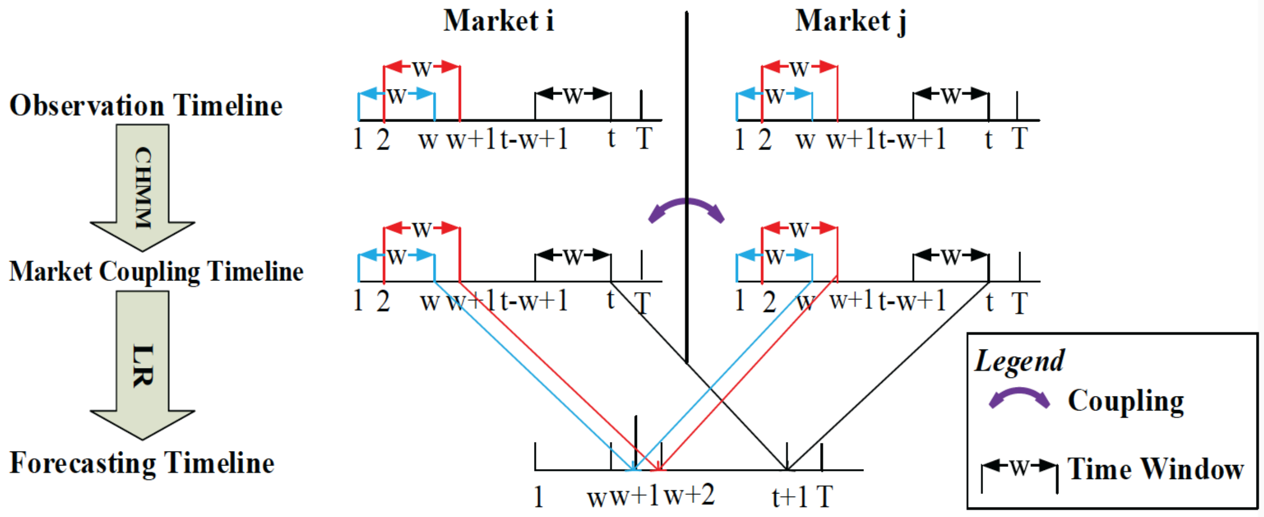


Fig. 3: Forecasting Process

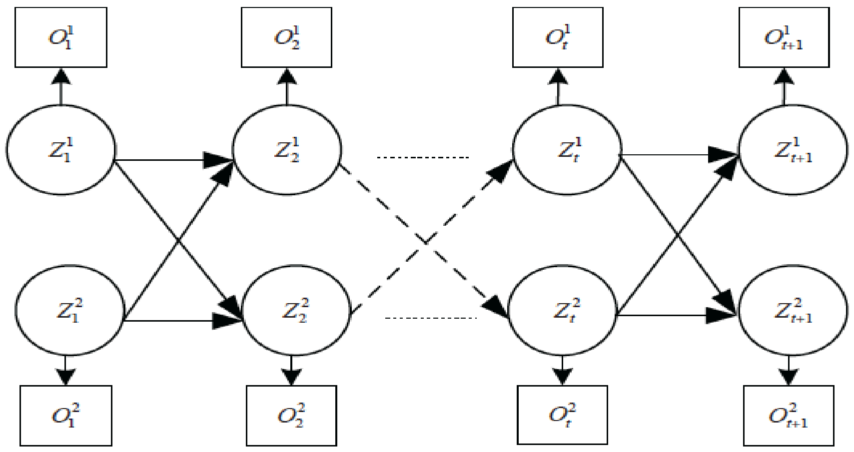
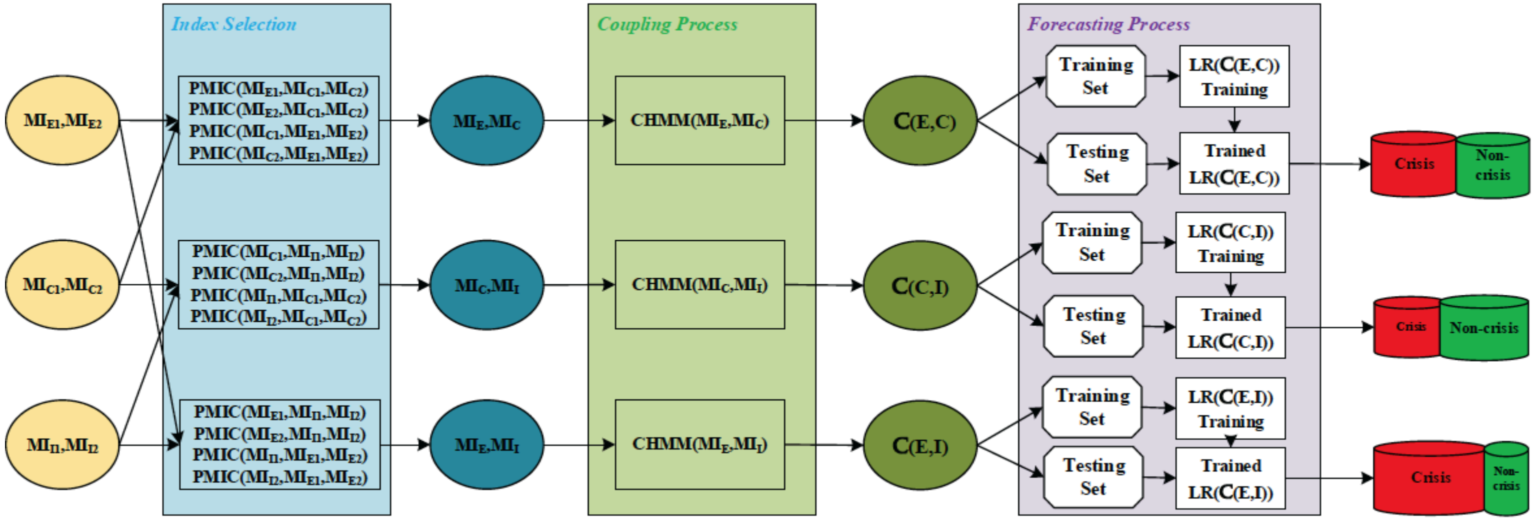


Fig. 1: A CHMM with Two Chains

- Temporal/periodical transition
- State transition within a market
- Coupling between markets
- CHMM for both transitions/influence within and between sequences

Modeling within/between-market couplings



- CHMM-LR:
  - (1) couplings between equity market and commodity market ($\mathbb{C}(E,C)$);
  - (2) couplings between equity market and interest market ($\mathbb{C}(E,I)$);
  - (3) couplings between commodity market and interest market ($\mathbb{C}(C,I)$).

- Select financial variables
- Modeling their within/between sequence couplings
- Forecasting

Case study: financial crisis

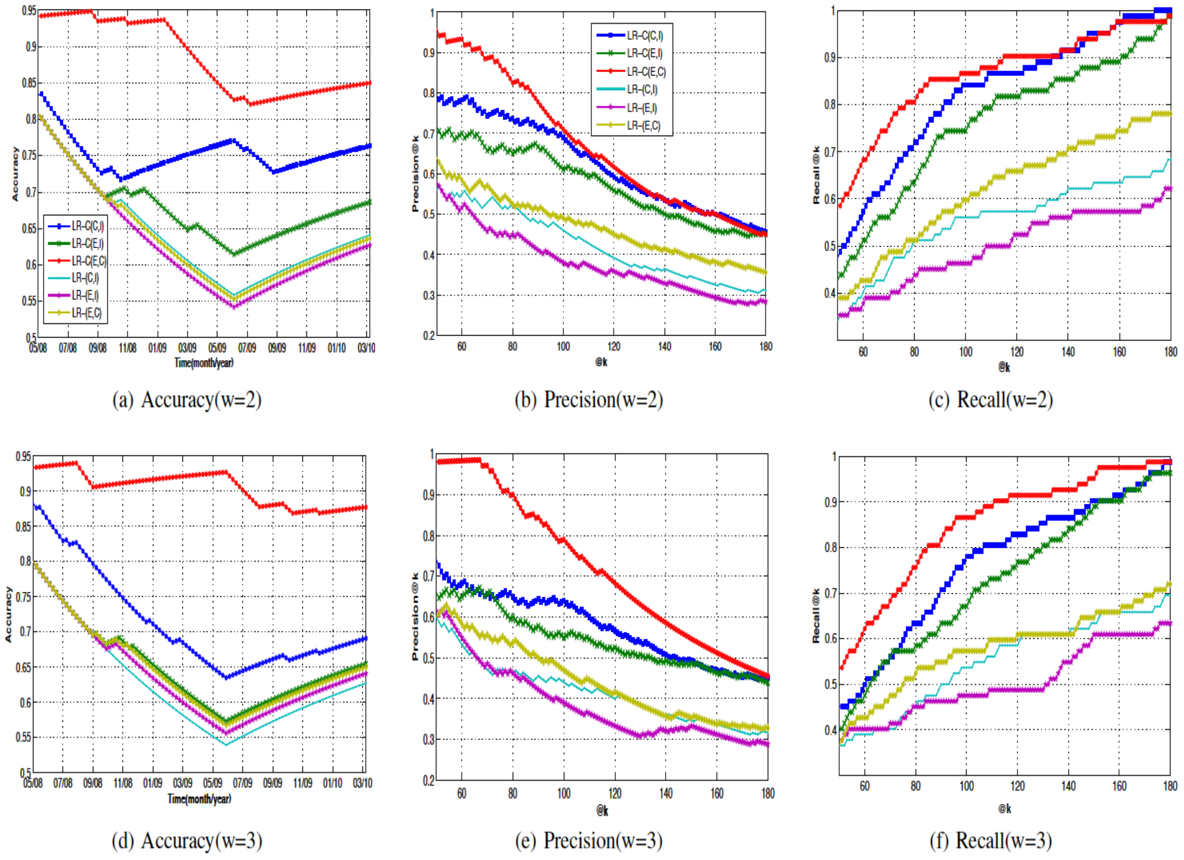


Fig. 7: Technical Performance of Various Approaches

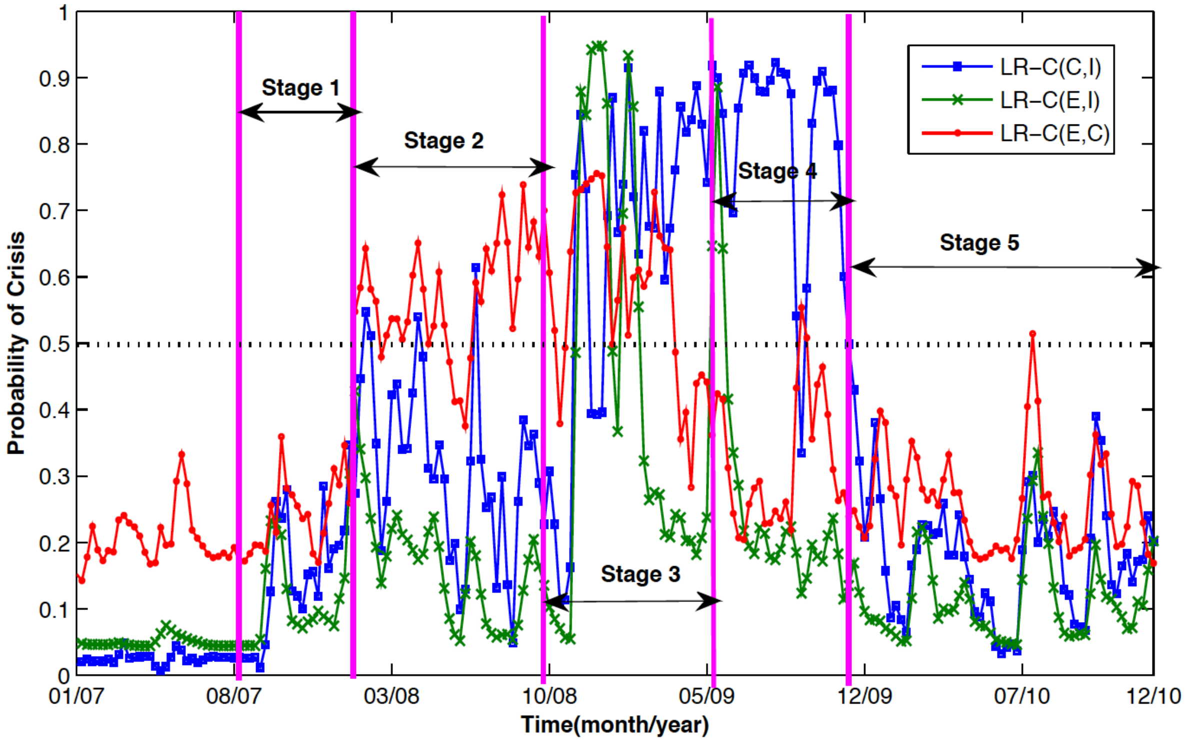


Fig. 8: Market Couplings Behavior during 2008 Financial Crisis ($w = 2$)

TABLE II: Selected Indicators

Pairwise Coupling	Market Indicator
$\mathbb{C}(E, C)$	E: DJIA /C:WTI Oil Price
$\mathbb{C}(C, I)$	C: Gold Price/C:TED Spread
$\mathbb{C}(E, I)$	E: DJIA /I:BAA Spread

More examples:
Deep Multivariate Coupling Learning

Deep financial representation

$$v_v = W_v([h_n; x_s])$$

$$h_w^t = LSTM(h_w^{t-1}, x_w^t)$$

$$N^{l+1} = \sigma(\sum_r D_r^{-\frac{1}{2}} A_r D_r^{-\frac{1}{2}} H^l W_r^l + W_h H^l)$$

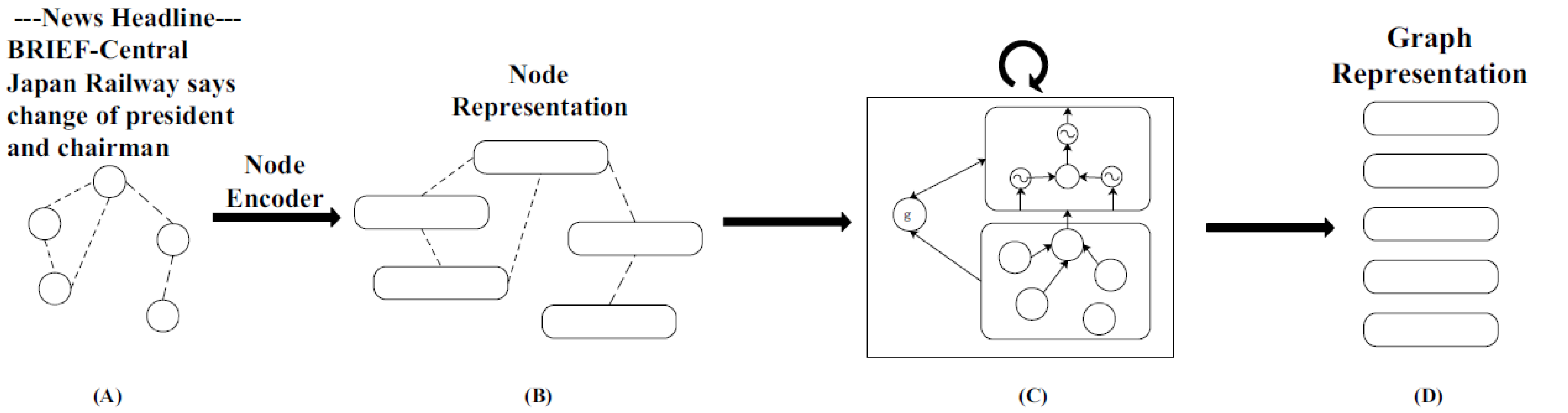
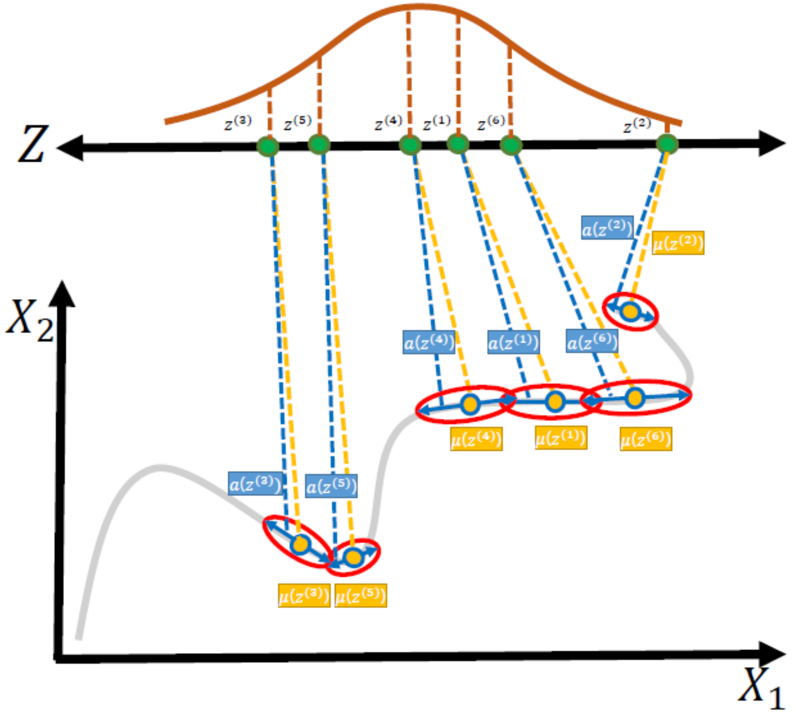


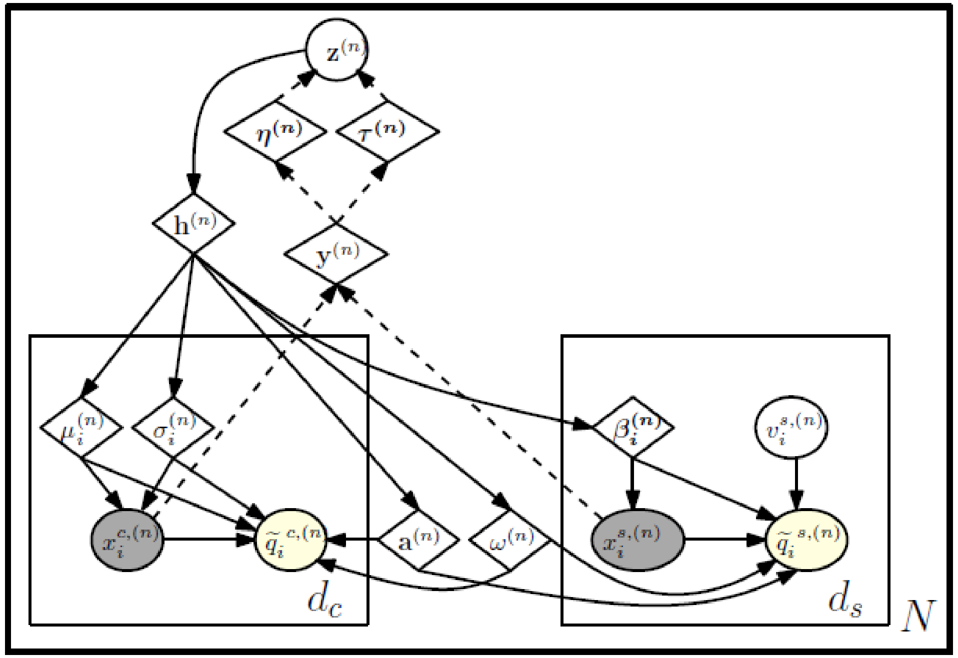
Figure 1: A brief description of our proposed LSTM-RGCN model. Each node in the graph represents one stock. A node can be attached with none or several overnight news text. The dashed lines indicate the two relations that connects stocks (**A**). The news is first encoded with the node feature encoder (**B**). Then the node embedding is fed into our proposed LSTM-RGCN model to make use of the correlation graph structure (**C**). Note that LSTM-RGCN can have multiple layers. Finally, the node vectors are used to predict the overnight stock price movement (**D**).

W Li et al. Modeling the Stock Relation with Graph Network for Overnight Stock Movement Prediction, IJCAI20

Neural-dependence coupled networks

Copula variational autoencoder for mixed data





$$\mathcal{F}(\theta, \phi; \boldsymbol{x}^{(n)}) = \mathbb{E}_{q_\phi(\boldsymbol{z}^{(n)}|\boldsymbol{x}^{(n)})}\left[\log p_\theta(\boldsymbol{x}^{(n)}|\boldsymbol{z}^{(n)})\right] - \mathrm{KL}\left[q_\phi(\boldsymbol{z}^{(n)}|\boldsymbol{x}^{(n)}) \,\|\, p_\theta(\boldsymbol{z}^{(n)})\right],$$

$$p_\theta(\boldsymbol{x}|\boldsymbol{z}) = p_\theta(\boldsymbol{x}^c, \boldsymbol{x}^s|\boldsymbol{z}) = c_\Psi(\cdot)\prod_{i=1}^{d_c}\mathcal{N}(\mu_i, \sigma_i^2)\prod_{i=1}^{d_s}\prod_{j=1}^{J}\beta_{i,j}^{\mathbb{I}(x_i^s=j)},$$

S. Suh and S Choi. Gaussian Copula Variational Autoencoders for Mixed Data

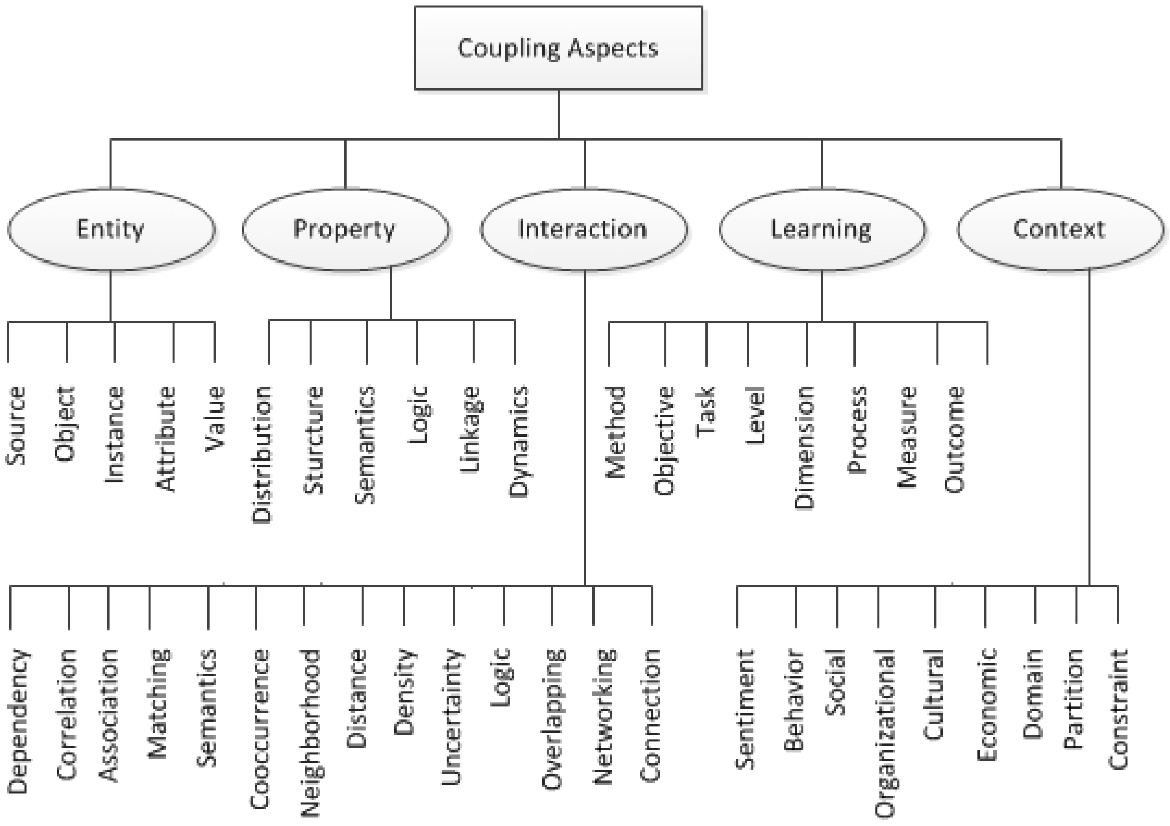
P. Wang and W. Wang. Neural Gaussian Copula for Variational Autoencoder

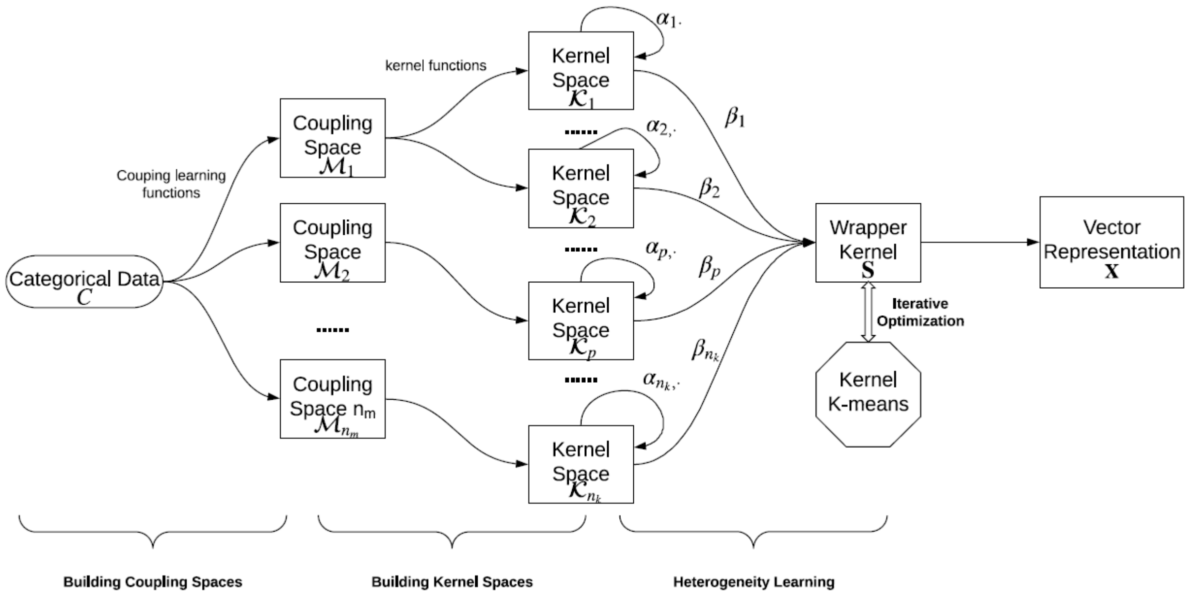
Concluding remarks

Coupling complexities in finance

- Macro/micro-variables across markets
- Structured/unstructured variables
- Within/between variable couplings
- Couplings: dependence, correlation, hidden relation, etc.
- Nonstationary, heterogeneous, multiscale, stylistic, …

Heterogenous and hierarchical couplings



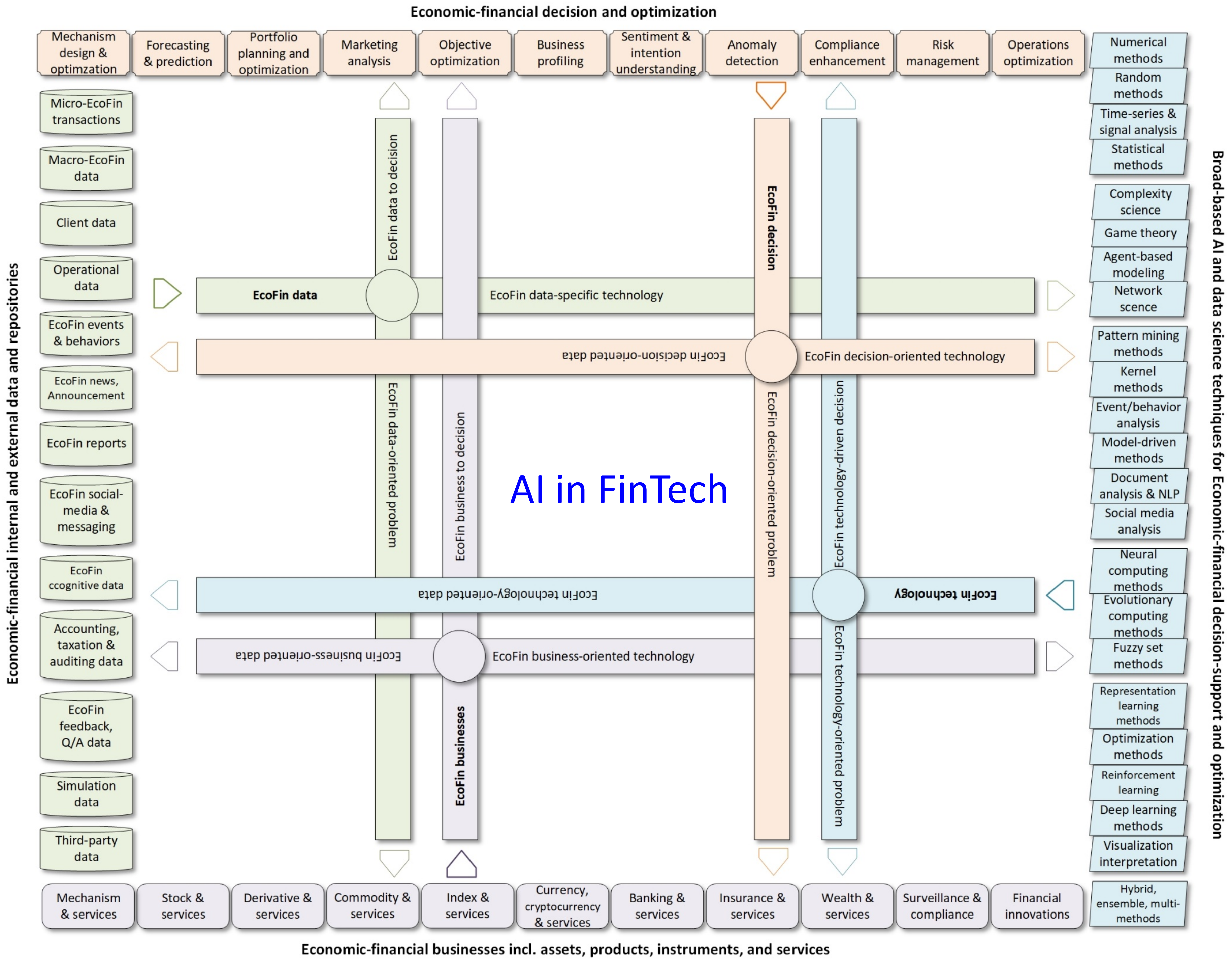


L Cao. Coupling Learning of Complex Interactions
www.datasciences.org

C Zhu et al. Unsupervised Heterogeneous Coupling Learning for Categorical Representation

Some recent work on AI in FinTech

- IJCAI2020 Special track on AI in FinTech
  https://www.ijcai.org/Proceedings/2020/
- IEEE Intelligent Systems special issues on AI and FinTech
  https://ieeexplore.ieee.org/document/9090124
- DSAA2020 journal track on Data Science and AI in FinTech
  dsaa2020.dsaa.co
- L Cao. AI in FinTech: A Research Agenda
  https://arxiv.org/abs/2007.12681
- L Cao. AI in Finance: A Review
  https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3647625



Thank you

L Cao.
AI in Finance: A Review

www.datasciences.org